CLASS-7

LESSON-8 RATIONAL NUMBERS

(This PDF Based on NCERT Book)

RATIONAL NUMBER(परिमेय संख्या)-A **rational number** is any number that can be expressed as a **fraction** or ratio of two integers, where the denominator is not zero.

Examples

- **Whole numbers and integers:** Any integer, like 5, can be written as the fraction $\frac{5}{1}$, so it's a rational number.
- **Fractions:** Simple fractions like $\frac{1}{2}$ or $-\frac{3}{4}$ are rational numbers by definition.
- **Terminating decimals:** Decimals that end, such as 0.75, can be written as a fraction ($\frac{3}{4}$)
- **Repeating decimals:** Decimals that have a repeating pattern, like 0.333..., can be written as a fraction ($\frac{1}{3}$).

POSITIVE AND NEGATIVE NUMBERS(धनात्मक और ऋणात्मक संख्या)-

Positive numbers are all numbers greater than zero. They are typically written without a sign in front of them, but can also be preceded by a plus sign (+).

Example- $\frac{1}{2}, \frac{3}{4}, \frac{1}{3}$

Negative numbers are all numbers less than zero. They are always written with a minus sign (-) in front of them.

Example- $-\frac{1}{2}, -\frac{3}{4}, -\frac{1}{3}$

RATIONAL NUMBERS ON A NUMBER LINE(संख्या रेखा पर परिमेय संख्या)-

Representing rational numbers on a number line involves visualizing their position between two integers. Here's a simple process to do that:

1. Identify the Integers

First, determine which two consecutive integers the rational number lies between.

- For a positive rational number like $\frac{2}{3}$, it will be between 0 and 1.

- For a negative rational number like $-\frac{5}{4}$,you can convert it to a mixed number first: $-1\frac{1}{4}$. This tells you it lies between -1 and -2.

2. Divide the Interval

The **denominator** of the fraction tells you how many equal parts to divide the interval between the two integers.

- For $\frac{2}{3}$, the denominator is 3, so you divide the space between 0 and 1 into three equal parts.
- For $-\frac{5}{4}$,the denominator is 4, so you divide the space between -1 and -2 into four equal parts.

3. Locate the Point

The **numerator** tells you how many of those equal parts to count from the starting integer.

- For a positive number, you count to the **right**. For $\frac{2}{3}$,you count two parts to the right from 0.
- For a negative number, you count to the **left**. For $-\frac{5}{4}$, you count one part to the left from -1.

This method helps to visually represent that rational numbers, which are ratios of two integers, fill in the spaces between whole numbers on a number line.

RATIONAL NUMBERS IN STANDRAD FROM(मानक रूप मे परिमेय संख्या)-

A rational number is in its **standard form** when it meets two specific conditions:

1. The **denominator is a positive integer**.
2. The **numerator and the denominator are coprime**, meaning their only common factor is 1. This is also called being in "lowest terms" or "simplest form."

How to Convert a Rational Number to Standard Form

If a rational number is not in standard form, you can convert it by following these steps:

1. **Make the denominator positive.** If the denominator is negative, multiply both the numerator and the denominator by -1.

- Example: $\frac{3}{-5} \rightarrow -\frac{3\times -1}{-5\times -1} = \frac{-3}{5}$

2. **Simplify the fraction.** Divide both the numerator and the denominator by their **greatest common divisor (GCD)**.
   - Example: To simplify $\frac{12}{18}$, find the GCD of 12 and 18, which is 6.
   - $\frac{12\div 6}{18\div 6} = \frac{2}{3}$ Since the numerator (2) and denominator (3) have no common factor other than 1, the fraction is now in standard form.

Combining both steps:

- Convert $\frac{10}{-25}$ to standard form.
- **Step 1:** Make the denominator positive: $\frac{10\times -1}{-25\times -1} = \frac{-10}{25}$.
- **Step 2:** Find the GCD of 10 and 25, which is 5.
- **Step 3:** Divide by the GCD: $\frac{-10\div 5}{25\div 5} = \frac{-2}{5}$
  - The standard form of $\frac{10}{-25}$ is $\frac{-2}{5}$.

COMPARISON OF RATIONAL NUMBERS(परिमेय संख्याओं की तुलना)-

Comparing rational numbers involves determining which number is greater than (>, less than (<, or equal to (=) another. Here are three common methods for comparing rational numbers:

1. Same Denominator Method

This is the most straightforward method. If the two rational numbers have the same denominator, you only need to compare their numerators.

- **For positive numbers:** The rational number with the **larger numerator** is the greater number.
- Example: Compare $\frac{5}{7}$ and $\frac{3}{7}$. Since 5>3, it follows that $\frac{5}{7} > \frac{3}{7}$.
- **For negative numbers:** The rational number with the **smaller numerator** is the greater number. This is because on a number line, a smaller negative number (closer to zero) is a greater value.
  - Example: Compare $-\frac{5}{7}$ and $-\frac{3}{7}$. Since −3>−5, it follows that $-\frac{3}{7} .> -\frac{5}{7}$.

2. Common Denominator Method

When the rational numbers have different denominators, you can convert them to equivalent fractions with a **common denominator** before comparing.

1. Find the **least common multiple (LCM)** of the denominators.
2. Convert each rational number to an equivalent fraction with this LCM as the new denominator.

3. Compare the numerators of the new fractions, as in the first method.

- Example: Compare $\frac{3}{4}$ and $\frac{5}{6}$.
    1. The LCM of 4 and 6 is 12.
    2. Convert the fractions: $\frac{3}{4}=\frac{3\times3}{4\times3}=\frac{9}{12}$ and $\frac{5}{6}=\frac{5\times2}{6\times2}=\frac{10}{12}$.
    3. Compare the numerators: 10>9, so $\frac{10}{12}>\frac{9}{12}$.Therefore, $\frac{5}{6}>\frac{3}{4}$.

RATION NUMBERS BETWEEN TWO RATIONAL NUMBERS(दो परिमेय संख्याओं के बीच परिमेय संख्या)-

There are an **infinite number** of rational numbers between any two rational numbers. You can find them using a few common methods.

Method 1: The Common Denominator Method

This is the most common way to find a specific number of rational numbers between two others.

1. **Find a common denominator.** If the denominators are different, find the least common multiple (LCM). If they are already the same but there are no whole numbers between the numerators, you'll need to create a larger common denominator.
2. **Create a "gap."** To create a larger gap between the numerators, multiply the numerator and denominator of both fractions by a number slightly larger than the number of rational numbers you want to find. For example, if you want to find 5 rational numbers, multiply by 6 (or 10, or 100).
3. **List the numbers.** Once the denominators are the same, list all the fractions with the same denominator and numerators that fall between the new numerators.

Example: Find 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

- The denominators are the same, but there's no whole number between 3 and 4.
- To create a gap, multiply both by 6:
    - $\frac{3}{5}\times\frac{6}{6}=\frac{18}{30}$
    - $\frac{4}{5}\times\frac{6}{6}=\frac{24}{30}$
- Now, list the rational numbers between $\frac{18}{30}$ and $\frac{24}{30}$:
    - $\frac{19}{30},\frac{20}{30},\frac{21}{30},\frac{22}{30},\frac{23}{30}$

Method 2: The Mean (Average) Method

This method finds one rational number that is exactly in the middle of the two numbers. You can repeat this process to find more.

1. **Add the two rational numbers.**

2. **Divide the sum by 2** (which is the same as multiplying by $\frac{1}{2}$).

Example: Find a rational number between $\frac{1}{2}$ and $\frac{3}{4}$.

1. Add the numbers: $\frac{1}{2}+\frac{3}{4}=\frac{2}{4}+\frac{3}{4}=\frac{5}{4}$
2. Divide by 2: $\frac{5}{4}\div 2=\frac{5}{4}\times\frac{1}{2}=\frac{5}{8}$
    - So, $\frac{5}{8}$ is a rational number between $\frac{1}{2}$ and $\frac{3}{4}$. You could then find a number between $\frac{1}{2}$ and $\frac{5}{8}$ by repeating the process.

OPERATIONS ON RATIONAL NUMBERS(परिमेय संख्याओं पर संक्रियाएं)-

Performing operations on rational numbers (addition, subtraction, multiplication, and division) is similar to performing these operations on fractions. The key is to handle the numerators and denominators correctly.

1. Addition and Subtraction

To add or subtract rational numbers, you must have a **common denominator**.

- **Same Denominators:** If the denominators are already the same, simply add or subtract the numerators and keep the denominator.
    - Example: $\frac{3}{8}+\frac{1}{8}=\frac{3+1}{8}=\frac{4}{8}=\frac{1}{2}$
    - Example: $\frac{5}{6}-\frac{1}{6}=\frac{5-1}{6}=\frac{4}{6}=\frac{2}{3}$
- **Different Denominators:** If the denominators are different, find the **least common multiple (LCM)** of the denominators. This LCM will be your new common denominator. Then, convert each fraction to an equivalent fraction with the new denominator and proceed as above.
    - Example: Add $\frac{1}{3}$ and $\frac{1}{4}$.
        1. The LCM of 3 and 4 is 12.
        2. Convert fractions: $\frac{1}{3}=\frac{1\times 4}{3\times 4}=\frac{4}{12}$ and $\frac{1}{4}=\frac{1\times 3}{4\times 3}=\frac{3}{12}$.
        3. Add the new fractions: $\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$.

2. Multiplication

To multiply rational numbers, you just multiply the numerators together and the denominators together. You can simplify before or after you multiply.

- Example: $\frac{2}{3}\times\frac{5}{7}=\frac{2\times 5}{3\times 7}=\frac{10}{21}$

- Example (with negative numbers): $-\frac{3}{4} \times \frac{2}{5} = \frac{-3 \times 2}{4 \times 5} = \frac{-6}{20} = -\frac{3}{10}$

3. Division

To divide one rational number by another, you multiply the first number by the **reciprocal** of the second number. The reciprocal is found by flipping the fraction (swapping the numerator and denominator).

- Example: Divide $\frac{3}{4}$ by $\frac{1}{2}$.
    1. The reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$.
    2. Multiply: $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2}$.